TP10

13

CALCULO I - AULA 65

084.F

$r_1 \equiv$ $A(0;-4)$, $B(6;8)$ $\Rightarrow m = \dfrac{8+4}{6-0} = \dfrac{12}{6} = \boxed{2}$

$\boxed{m_1 = 2}$

$y = m(x-x_0)+y_0$

$y = 2(x-6)+8 \Rightarrow y = 2x-12+8$

$r_1 \equiv \boxed{y = 2x-4}$

$r_2 \equiv$ $A(0;-4)$, $C(9;5)$ $\Rightarrow m_2 = \dfrac{5+4}{9} = 1 \Rightarrow \boxed{m_2 = 1}$

$y = 1(x-0)-4 \Rightarrow r_2 \equiv \boxed{y = x-4}$

$r_3 \equiv$ $A(6;8)$

$y = -1(x-6)+8 \Rightarrow y = -x+6+8$

$r_3 \equiv \boxed{y = -x+14}$

$$\text{AREA} = \int_0^6 \left[(2x-4)-(x-4)\right]dx + \int_6^9 \left[(-x+14)-(x-4)\right]dx$$

$$\text{AREA} = \int_0^6 (2x-x-\cancel{4}+\cancel{4})\,dx + \int_6^9 (-x+14-x+4)\,dx$$

$$\text{AREA} = \int_0^6 x\,dx + \int_6^9 (-2x+18)\,dx$$

$$A = \left.\frac{x^2}{2}\right]_0^6 + \left.\left(-\frac{\cancel{2}x^2}{\cancel{2}}+18x\right)\right]_6^9$$

$$A = \frac{\overset{18}{\cancel{36}}}{\cancel{2}} + (-81+18\cdot 9)-(-36+18\cdot 6)$$

$$A = 18-81+162+36-108$$

$$\boxed{A = 27}$$

20 - Estudia y representa la curva $y = \dfrac{(x-1)^2}{(x+1)^2}$. Encuentra el área comprendida entre la curva, el eje x y las rectas $x=1$ y $x=2$

$$y = \frac{(x-1)^2}{(x+1)^2}$$

AV : $x = -1$

AH : $y = 1$

$\cap$ con el eje y : $y = 1$

$\cap$ con el eje x : $x = 1$

Df : $\mathbb{R} - \{-1\}$

Paridad no tiene

$$f'(x) = \frac{2(x-1)\cdot(x+1)^2 - (x-1)^2 \cdot 2(x+1)}{(x+1)^4} = \frac{(x-1)\cancel{(x+1)}(2x+2-2x+2)}{(x+1)^{\cancel{4}\,3}}$$

$$f'(x) = \frac{4(x-1)}{(x+1)^3} = 0 \Rightarrow 4(x-1) = 0 \Rightarrow \boxed{x=1} \text{ PC.}$$

$(-\infty;-1)$ $(-1;1)$ $(1;+\infty)$

-1 1

$\boxed{f(1) = 0}$

$f'(-2) > 0$ $f'(0) < 0$ $f'(2) > 0$

CRECE DECRECE CRECE

mínimo en $x = 1$

$\downarrow$

$(1;0)$

$$f'(x) = \frac{4(x-1)}{(x+1)^3}$$

$$f''(x) = \frac{4(x+1)^3 - 4(x-1)\cdot 3(x+1)^2}{(x+1)^6}$$

$$f''(x) = \frac{4\cancel{(x+1)^2}\left[(x+1) - 3(x-1)\right]}{(x+1)^{\cancel{6}\,4}}$$

$$f''(x) = \frac{4(-2x+4)}{(x+1)^4} = \frac{4\cdot(-2)(x-2)}{(x+1)^4} = 0 \Rightarrow x = 2 \text{ posible PI}$$

$(-\infty;-1)$ $(-1;2)$ $(2;+\infty)$

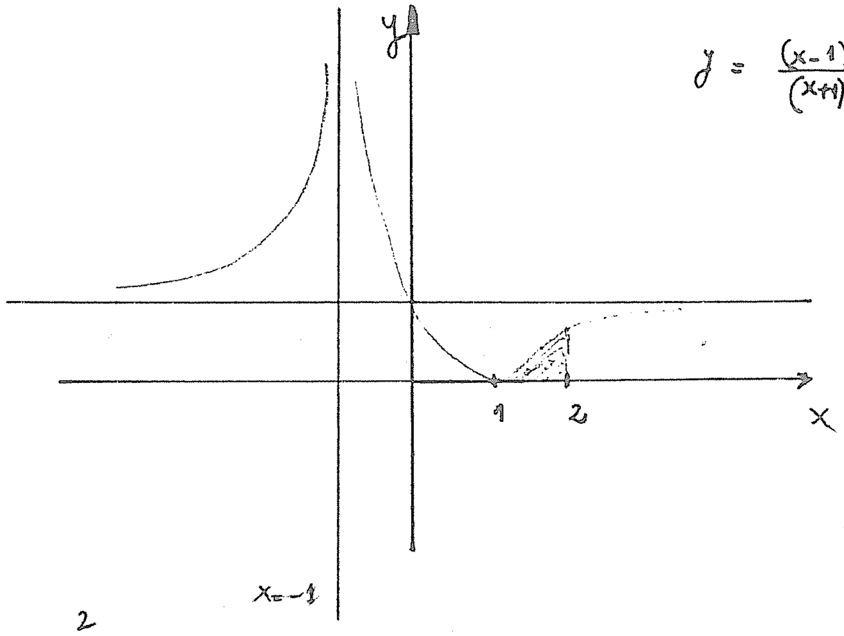
$\cup$ -1 $\cup$ 2 $\cap$

$f''(-2) > 0$ $f''(0) > 0$ $f''(3) < 0$

$\left(2; \frac{1}{9}\right)$ PI

TP 10

14|



$$y = \frac{(x-1)^2}{(x+1)^2}$$

$$y = 1$$

$$A = \int_1^2 \frac{(x-1)^2}{(x+1)^2}\, dx =$$

$(x-1)^2 = x^2 - 2x + 1$

$(x+1)^2 = x^2 + 2x + 1$

$$\begin{array}{l|l} \cancel{x^2} - 2x + 1 & x^2 + 2x + 1 \\ \hline -\cancel{x^2} - 2x - 1 & 1 \\ \hline -4x & \end{array}$$

$$\frac{(x-1)^2}{(x+1)^2} = 1 + \frac{(-4x)}{(x+1)^2}$$

— x —

$$\frac{-4x}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\frac{-4x}{(x+1)^2} = \frac{A(x+1) + B}{(x+1)^2}$$

$$-4x = Ax + A + B$$

$$A = -4$$

$$A + B = 0 \Rightarrow -4 + B = 0 \Rightarrow \boxed{B = 4}$$

— x —

$$A = \int_1^2 \frac{(x-1)^2}{(x+1)^2}\, dx = \int_1^2 \left(1 + \frac{(-4x)}{(x+1)^2}\right) dx$$

$$A = \int_1^2 \left(1 - \frac{4}{x+1} + \frac{4}{(x+1)^2}\right) dx$$

$$A = x - 4\ln|x+1| - 4(x+1)^{-1}\Big]_1^2$$

$$A = 2 - 4\ln 3 - 4\cdot\frac{1}{3} - \left(1 - 4\ln 2 - 4\cdot\frac{1}{2}\right)$$

$$A = 2 - 4\ln 3 - \frac{4}{3} - 1 + 4\ln 2 + 2$$

$$A = \frac{5}{3} + \ln\frac{16}{81}$$

21.- Estudia y representa la curva $y = x^2 \ln x$. Encuentra el área comprendida entre la curva, el eje x la recta $x = 1$ y la recta $x = 2$

$y = x^2 \ln x$ $\qquad$ Df: $\mathbb{R}^+$

A.V.

$$\lim_{x\to 0^+} x^2 \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = 0 \qquad \nexists \text{ AV.} \quad \nexists \text{ AH}$$

$$y' = 2x \cdot \ln x + x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} = 2x \ln x + x = x\,(2\ln x + 1)$$

$$y' = x\,(2\ln x + 1) = 0 \Rightarrow 2\ln x + 1 = 0$$

$$\ln x = -\frac{1}{2} \Rightarrow x = e^{-1/2}$$

$(0; e^{-1/2})$ $\qquad$ $(e^{-1/2}; +\infty)$

$0 \qquad e^{-1/2}$

$$f'(e^{-3}) = e^{-1/3}\,(2 \ln e^{-3} + 1)$$

$$f'(e^{-3}) = e^{-1/3}\,(-6 \underbrace{\ln e}_{1} + 1) < 0$$

$f'(e^{-3}) < 0$ $\qquad$ $f'(e) > 0$

decrece $\qquad$ crece

mínimo $\left(e^{-1/2};\ f(e^{-1/2})\right) = \left(e^{-1/2};\ e^{-1}\left(-\frac{1}{2}\right)\right)$

$$y' = x\,(2\ln x + 1)$$

$$y'' = 2\ln x + 1 + \cancel{x} \cdot 2 \cdot \frac{1}{\cancel{x}} \Rightarrow y'' = 2\ln x + 3 = 0$$

$$2 \ln x = -3$$

$$\ln x = -\frac{3}{2}$$

$$x = e^{-3/2}$$

Concavidad :

$(0; e^{-3/2})$ $\qquad$ $(e^{-3/2}; +\infty)$

$0 \qquad e^{-3/2}$

$f''(e^{-3}) < 0$ $\qquad$ $f''(e) > 0$

$\cap$ $-$ $\qquad$ $\cup$ $+$

Pi $\left(e^{-3/2};\ f(e^{-3/2})\right) = \left(e^{-3/2};\ e^{-3}\left(-\frac{3}{2}\right)\right)$.

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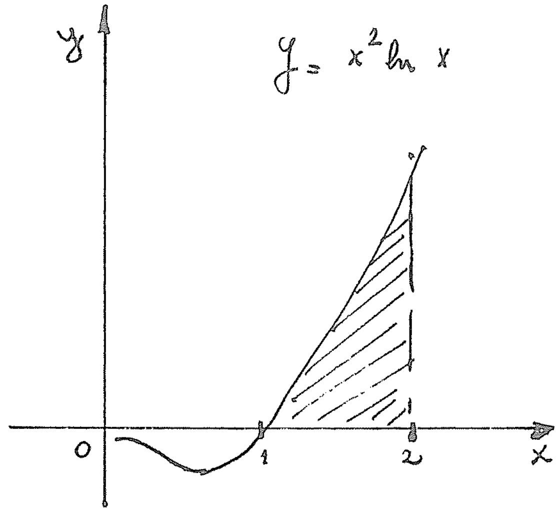
15/



$A = \int_1^2 x^2 \ln x \, dx =$

$u = \ln x$

$du = \frac{1}{x} dx$

$dv = x^2$

$v = \frac{x^3}{3}$

Resolvemos por Partes:

$A = \int_1^2 x^2 \ln x \, dx = (\ln x) \cdot \frac{x^3}{3} - \int \frac{\cancel{x^3}^{2}}{3} \frac{1}{\cancel{x}_{1}} dx$

$A = \int_1^2 x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 \, dx$

$\int_1^2 x^2 \ln x \, dx = \left(\frac{1}{3} x^3 \ln x - \frac{1}{3} \frac{x^3}{3} \right) \Big]_1^2$

$\int_1^2 x^2 \ln x \, dx = \left(\frac{1}{3} 2^3 \ln 2 - \frac{1}{9} 2^3 \right) - \left(\frac{1}{3} 1^3 \underbrace{\ln 1}_{0} - \frac{1}{9} 1^3 \right)$

ÁREA $= \frac{8}{3} \ln 2 - \frac{8}{9} + \frac{1}{9}$

ÁREA $= \frac{8}{3} \ln 2 - \frac{7}{9}$

22.- Dada la función $f(x) = x\sqrt{3-2x}$ determina su dominio, intervalos de crecimiento y decrecimiento y calcula el área de la región delimitada por la gráfica de la función el eje x correspondiente al intervalo $[0;3/2]$.

$f(x) = x\sqrt{3-2x}$

Df: $3-2x \geq 0$ $\quad -2x \geq -3$ $\quad \boxed{x \leq \frac{3}{2}}$

$(-\infty ; \frac{3}{2}]$

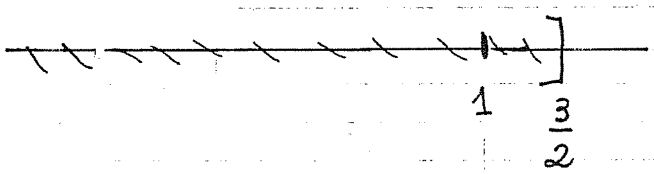
$$f'(x) = \sqrt{3-2x} + x \frac{1}{2\sqrt{3-2x}} \cdot (-2)$$

$$f'(x) = \frac{2(\sqrt{3-2x})^2 + 2x}{\sqrt{3-2x}}$$

$$f'(x) = \frac{2(3-2x) - 2x}{\sqrt{3-2x}} = \frac{6-4x-2x}{\sqrt{3-2x}}$$

$$f'(x) = \frac{6-6x}{\sqrt{3-2x}} = \frac{-6(x-1)}{\sqrt{3-2x}} = 0 \qquad x-1=0 \quad \boxed{x=1} \text{ Pc.}$$

$(-\infty ; 1)$ $\quad (1 ; \frac{3}{2})$

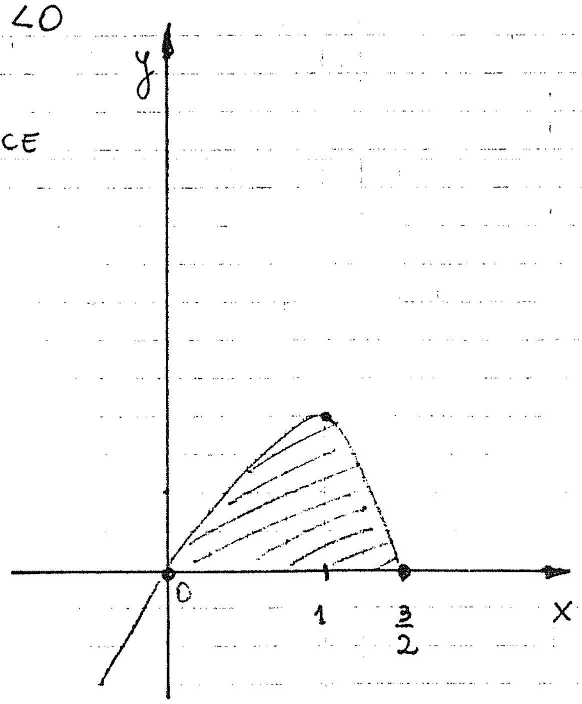


$f'(0) > 0$ $\quad f'(1,1) < 0$

CRECE $\quad$ DECRECE

Máximo en $x = 1$.

máximo : $(1;1)$



TP 10

16/ $\text{Area} = \int_0^{3/2} x\,\sqrt{3-2x}\,dx = \left(\frac{1}{10}\sqrt{(3-2x)^5} - \frac{1}{2}\sqrt{(3-2x)^3}\right)\Big]_0^{3/2}$

$$A = \left[\frac{1}{10}\sqrt{(\underbrace{3-2.\tfrac{3}{2}}_{0})^5} - \frac{1}{2}\sqrt{(\underbrace{3-2.\tfrac{3}{2}}_{0})^3}\right] - \left[\frac{1}{10}\sqrt{3^5} - \frac{1}{2}\sqrt{3^3}\right]$$

$$A \cong 1{,}04$$

— x —

Resolvemos :

$$\int x \overbrace{\sqrt{3-2x}}^{u}\,\overbrace{dx}^{-u\,du}$$

$x = \frac{3-u^2}{2}$

$u = \sqrt{3-2x}$

$u^2 = 3-2x$

$\frac{u^2-3}{-2} = x$

$\frac{3-u^2}{2} = x$

$-\frac{\cancel{2}u}{\cancel{2}}du = dx$

$-u\,du = dx$

Sustituyendo :

$$= \int \left(\frac{3-u^2}{2}\right).\,u\,(-u\,du)$$

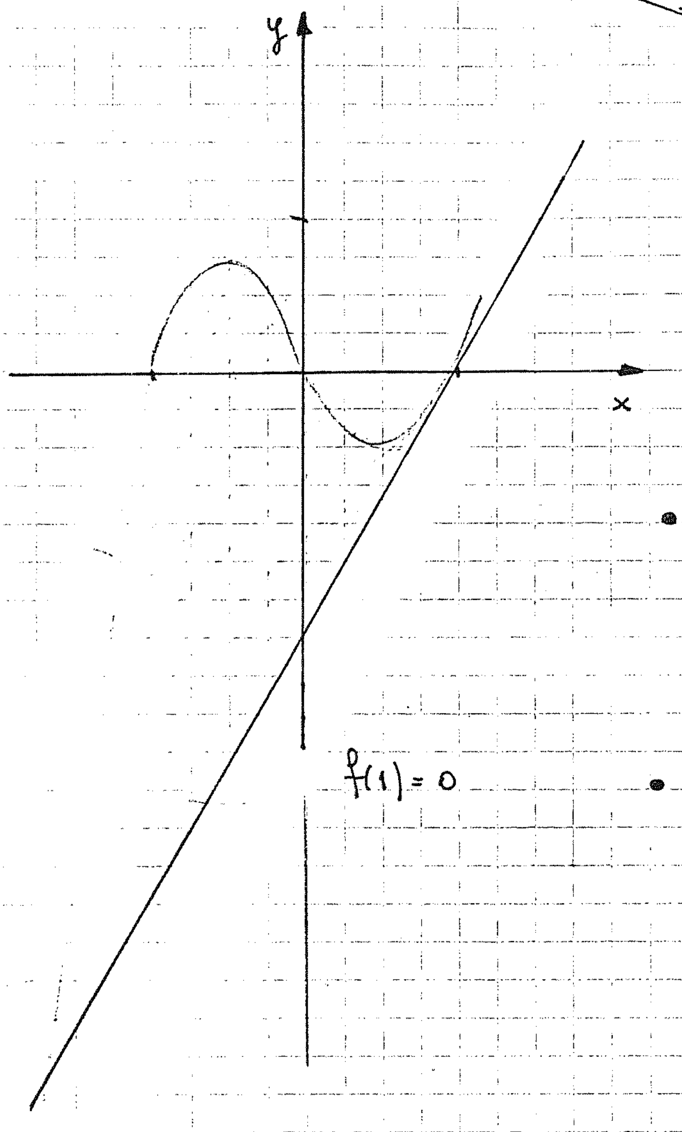
$$= \frac{1}{2}\int(-3u^2+u^4)\,du$$

$$= \frac{1}{2}\left[-\frac{\cancel{3}u^3}{\cancel{3}} + \frac{u^5}{5}\right]+C = -\frac{1}{2}\,(3-2x)\sqrt{3-2x} + \frac{1}{10}\sqrt{(3-2X)^5}$$

23 - Calcula el área de la región encerrada por la curva $y = x^3 - x$ y la recta tangente a esta curva en $x = 1$

$y = x^3 - x$ Df : $\mathbb{R}$.

ceros : $x(x^2-1) = 0$ → $x = 0$, $x = 1$, $x = -1$



$f(1) = 0$

- $y = x^3 - x$

$y' = 3x^2 - 1 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

- En $x = 1$ la pendiente de la recta tg es :

$f'(1) = 3 \cdot 1^2 - 1 = \boxed{2}$

- Recta tg en $x = 1$:

$y_t = f'(1)(x-1) + f(1)$

$y = 2(x-1) + 0$

$\boxed{y = 2x - 2}$ Recta tg

Encontramos los puntos donde la función y la recta tg. se cortan:

$x^3 - x = 2x - 2$

$x^3 - x - 2x + 2 = 0$

$\boxed{x^3 - 3x + 2 = 0} \Rightarrow \boxed{x = 1}$

$(x^3 + 0x^2 - 3x + 2) : (x - 1) = x^2 + x - 2$ → $\boxed{x = 1}$, $\boxed{x = -2}$

	1	0	-3	2
1	↓	1	1	-2
	1	1	-2	0 Resto

TP10

17)

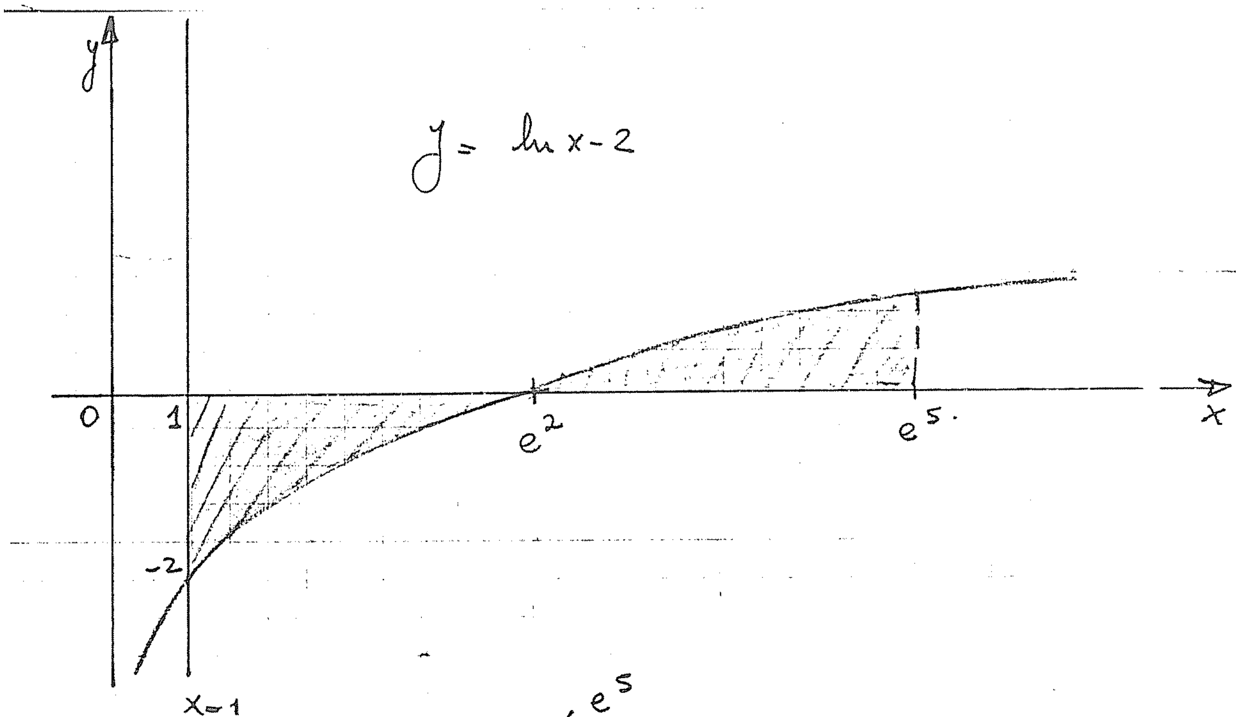
$$AREA = \int_{-2}^{1} (x^3 - x - 2x + 2)\, dx = \int_{-2}^{1} (x^3 - 3x + 2)\, dx$$

$$AREA = \left(\frac{x^4}{4} - \frac{3}{2} x^2 + 2x \right) \Big|_{-2}^{1}$$

$$AREA = \left(\frac{1^4}{4} - \frac{3}{2} 1^2 + 2.1 \right) - \left(\frac{(-2)^4}{4} - \frac{3}{2} (-2)^2 + 2.(-2) \right)$$

$$AREA = \frac{1}{4} - \frac{3}{2} + 2 - \cancel{4} + 6 \cancel{+4} = -\frac{5}{4} + 8 = \boxed{\frac{27}{4}}$$

24.- Calcula el área de la región encerrada por la curva $y = \ln x - 2$, $y = 0$, y las rectas $x = 1$ y $x = e^5$.



$$A = \int_{1}^{e^2} (2 - \ln x)\, dx + \int_{e^2}^{e^5} (2 - \ln x)\, dx$$

$$A = 2x - (x \ln x - x) \Big|_{1}^{e^2} + (x \ln x - x - 2x) \Big|_{e^2}^{e^5}$$

$$A = (3x - x \ln x) \Big|_{1}^{e^2} + (x \ln x - 3) \Big|_{e^2}^{e^5}$$

$$A = 3e^2 - e^2.2 - 3 + (e^5.5 - 3e^5) - (e^2.2 - 3e^2)$$

$$A = e^2 - 3 + 2.e^5 + e^2 \Rightarrow \boxed{A = 2e^2 + 2e^5 - 3}$$

25.- Estudia y representa la curva $y=(x-1).e^x$ Encuentra el área comprendida entre la curva, el eje x y las rectas $x=-1$ y $x=2$

$y = (x-1)\,e^x$ Df: $\mathbb{R}$

No tiene paridad $\cap$ con el eje x : $x = 1$

$\cap$ con el eje y : $y = -1$

AV = no tiene

AH :

$$\lim_{x\to-\infty} (x-1)\,e^x = \lim_{x\to-\infty} \frac{x-1}{e^{-x}}$$

$$= \lim_{x\to-\infty} \frac{1}{(-1)\,e^{-x}} = 0 \Rightarrow y=0 \text{ AH cuando } x\to-\infty.$$

$$y' = e^x + (x-1)\,e^x = e^x\,[\cancel{1} + x \cancel{-1}]$$

$$y' = e^x . x = 0 \Rightarrow \boxed{x=0} \text{ PC.}$$

$(-\infty;0)$ $(0;+\infty)$

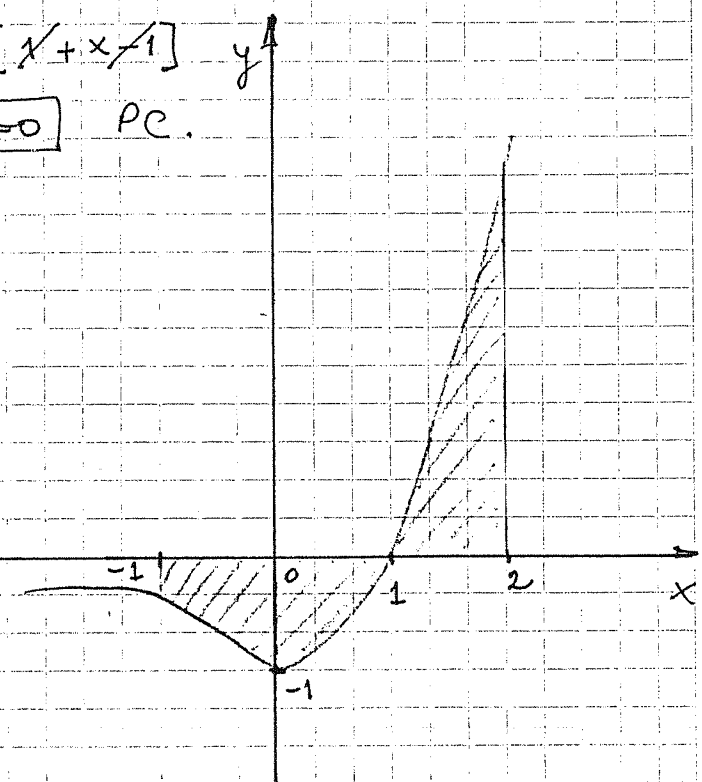
0

$f'(-1)<0$ $f'(1)>0$

decrece crece

mínimo: $(0, f(0))$

$(0;-1)$



$y' = e^x . x$

$y'' = e^x x + e^x$

$y'' = e^x\,(x+1) = 0$

$x+1 = 0$

$\boxed{x=-1}$

$(-\infty,-1)$ $(-1;+\infty)$

-1

$f''(-2)<0$ $f''(0)>0$

$-$ $+$

$P_i : (-1; -2\,e^{-1})$

$$A = \int_{-1}^{1} \left[-(x-1)\, e^x\right] dx + \int_{1}^{2} (x-1)\, e^x\, dx$$

$$A = \int_{-1}^{1} \left(-x e^x + e^x\right) dx + \int_{1}^{2} \left(x e^x - e^x\right) dx$$

$$A = \left[-e^x(x-1) + e^x\right]_{-1}^{1} + \left[e^x(x-1) - e^x\right]_{1}^{2}$$

$$A = \left[-e^1\underbrace{(1-1)}_{0} + e^1\right] - \left[-e^{-1}(-1-1) + e^{-1}\right] + \left[e^2(2-1) - e^2\right] - \left[e^1\underbrace{(1-1)}_{0} - e^1\right]$$

$$A = e + e^{-1}(-2) + e^{-1} + \cancel{e^2} - \cancel{e^2} + e = \boxed{2e - 3e^{-1}}$$

— x —

Resolvemos: $\int x e^x\, dx =$

x Partes
$u = x$
$du = dx$
$e^x\, dx = dv$
$e^x = v$

$$= x e^x - \int e^x\, dx$$

$$= x e^x - e^x + C$$

$$= e^x(x-1) + C$$

— x —

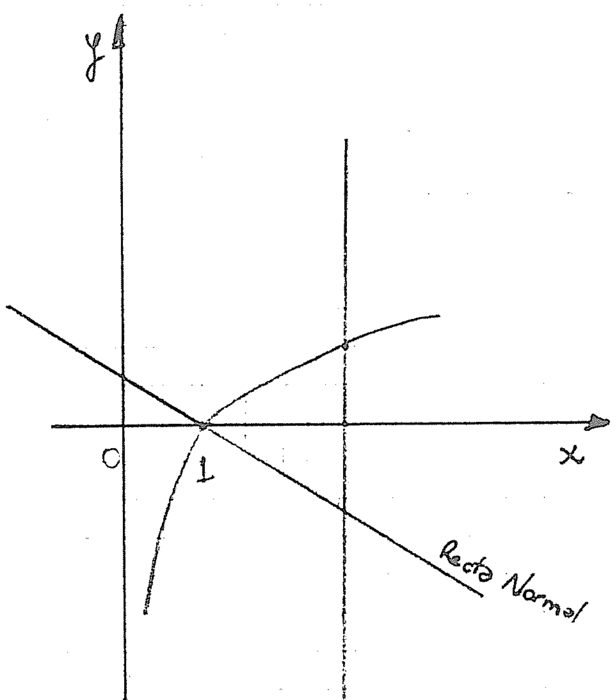
26.- Calcula el área de la figura delimitada por $f(x) = \ln x$, $x = e$ y la recta normal a $f(x)$ en (1;0).

$f(x) = \ln x$ $\qquad f(1) = 0$

$f'(x) = \frac{1}{x}$

$f'(1) = 1$ pendiente de la recta tg

$-\frac{1}{f'(1)} = -1$ pendiente de la recta normal



Recta normal: $y = -1(x-\ ) + 0$

$$\boxed{y = -x + 1}$$

$$A = \int_1^e (\ln x + x - 1)\, dx = x \ln x - x + \frac{x^2}{2} - x \Big]_1^e$$

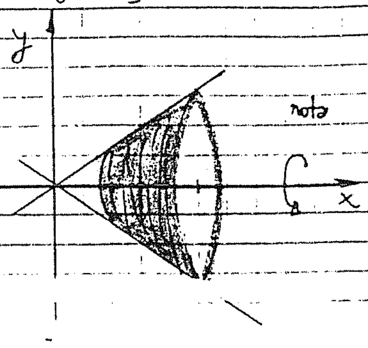
$$A = x \ln x - 2x + \frac{x^2}{2} \Big]_1^e = e - 2e + \frac{e^2}{2} - \left(-2 + \frac{1}{2}\right) \Rightarrow \boxed{A = -e + \frac{e^2}{2} + \frac{3}{2}}$$

27.- Halla los volúmenes de los sólidos de revolución engendrados cuando las curvas indicadas rotan alrededor del eje x

a) $y = 2x/3$ $\quad [2;5]$ $\qquad$ b) $y = x^2$ $\quad [1;2]$

c) $y = \operatorname{sen} x$ $\quad [0, \pi/2]$ $\qquad$ d) $y = x^2 + 3$ $\quad [-2;2]$

a) $y = \frac{2}{3}x \qquad \boxed{[2;5]}$



$$\text{Volumen} = \pi \int_a^b [f(x)]^2 \, dx$$

$$V = \pi \int_2^5 \left(\frac{2}{3}x\right)^2 dx$$

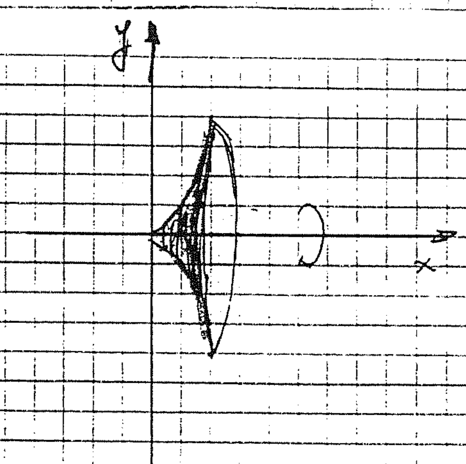
$$V = \frac{4}{9}\pi \int_2^5 x^2 \, dx$$

$$V = \frac{4}{9}\pi \left.\frac{x^3}{3}\right]_2^5$$

$$V = \frac{4}{9}\pi \left[\frac{5^3}{3} - \frac{2^3}{3}\right]$$

$$V = \frac{4}{9}\pi \, \frac{125-8}{3} \Rightarrow V = \frac{468}{27}\pi \approx \boxed{17{,}3\,\pi}$$

b) $y = x^2 \qquad [1;2]$

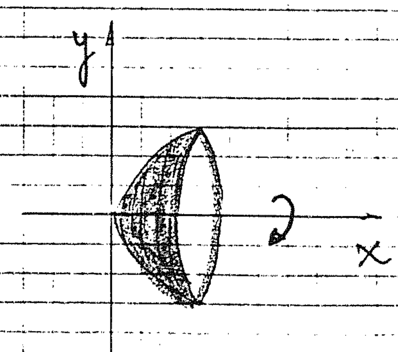


$$V = \pi \int_1^2 x^4 \, dx$$

$$V = \pi \left.\frac{x^5}{5}\right]_1^2 = \pi\left(\frac{32}{5} - \frac{1}{5}\right)$$

$$\boxed{V = \frac{31}{5}\pi \text{ u. de v.}}$$

c) $y = \operatorname{sen} x \qquad \boxed{[0; \frac{\pi}{2}]}$



$$V = \pi \int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x \, dx$$

$$V = \pi \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2x) \, dx$$

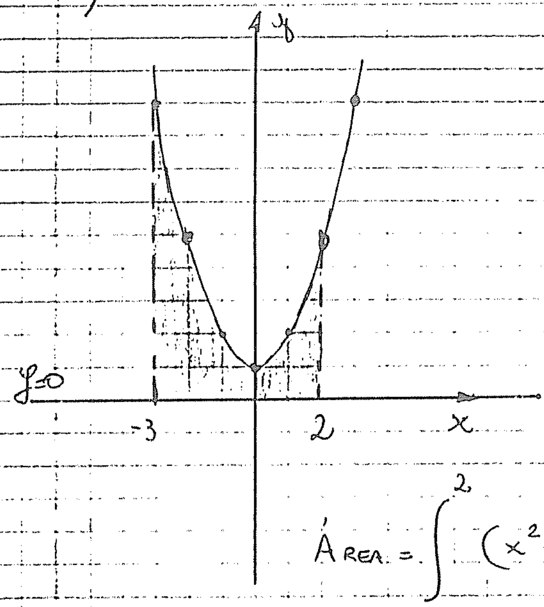
$$V = \pi \, \frac{1}{2} \left.\left(x - \frac{\operatorname{sen} 2x}{2}\right)\right]_0^{\frac{\pi}{2}}$$

$$V = \frac{\pi}{2}\left[\frac{\pi}{2} - \frac{\overbrace{\operatorname{sen} 2\pi/2}^{0}}{2}\right] \rightarrow \boxed{V = \frac{\pi^2}{4} \text{ unidades de volumen}}$$

TP Nº 10 CÁLCULO I

6) Hallar el área de las figuras limitadas por las curvas indicadas

a)



$$\begin{cases} y = x^2+1 \\ y=0 \\ x=-3 \\ x=2 \end{cases}$$

$$\text{ÁREA} = \int_{-3}^{2} (x^2+1)\,dx = \left(\frac{x^3}{3}+x\right)\Big]_{-3}^{2}$$

$$\text{ÁREA} = \left(\frac{2^3}{3}+2\right) - \left[\frac{(-3)^3}{3}+(-3)\right]$$

$$A = \frac{8}{3}+2-\left[-9-3\right] = \frac{8}{3}+2+12$$

$$A = \frac{8}{3}+14 = \frac{50}{3}$$

$$\boxed{A = \frac{50}{3}}$$

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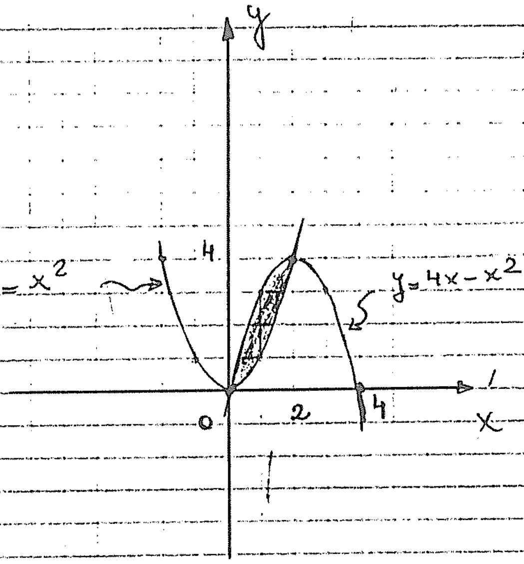
b) $\begin{cases} y = x^2 \\ y = 4x-x^2 \end{cases}$



$4x-x^2=0$

$x(4-x)=0$ — $x=0$, $x=4$

$x_v = \frac{-4}{2(-1)} = \boxed{2}$

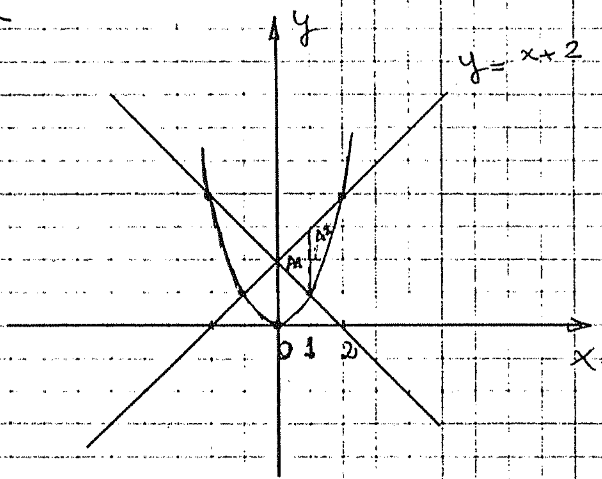
$y_v = 4.2-2^2$
$8-4=\boxed{4}$

$$\text{ÁREA} = \int_0^2 \left[(4x-x^2)-x^2\right]dx = \int_0^2 (4x-2x^2)\,dx$$

$$\text{ÁREA} = \left(\frac{4x^2}{2}-\frac{2x^3}{3}\right)_0^2 = \left(2.2^2-\frac{2}{3}2^3\right)-0 = 8-\frac{16}{3} = \boxed{\frac{8}{3}}$$

ÁREA

c) $\begin{cases} y = x^2 \\ y = x+2 \qquad 0 \le x \le 2 \\ y = -x+2 \end{cases}$



ÁREA = $A_1 + A_2$

ÁREA = $\int_0^1 \left[(x+2) - (-x+2)\right] dx + \int_1^2 \left[(x+2) - x^2\right] dx$

ÁREA = $\int_0^1 2x \, dx + \int_1^2 (x+2-x^2) \, dx$

$$A = \frac{\cancel{2}x^2}{\cancel{2}}\Big]_0^1 + \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big]_1^2$$

$$A = (1-0) + \left(\frac{2^{\cancel{2}}}{\cancel{2}} + 2.2 - \frac{2^3}{3}\right) - \left(\frac{1^2}{2} + 2.1 - \frac{1^3}{3}\right)$$

$$A = 1 + \left(6 - \frac{8}{3}\right) - \left(\frac{1}{2} + 2 - \frac{1}{3}\right)$$

$$1 + \frac{10}{3} - \frac{3+12-2}{6}$$

$$1 + \frac{10}{3} - \frac{13}{6} = \frac{6+20-13}{6} = \boxed{\frac{13}{6}}$$

$$\boxed{\text{ÁREA} = \frac{13}{6}}$$

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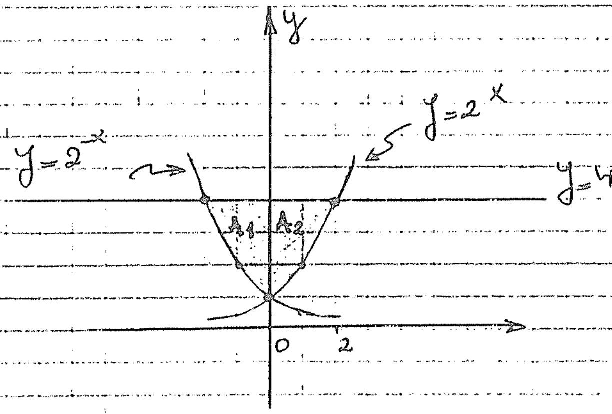
d) $\begin{cases} y = 2^{x} \\ y = 2^{-x} \\ y = 4 \end{cases}$



$A_1 = A_2 \Rightarrow A = 2A_2$

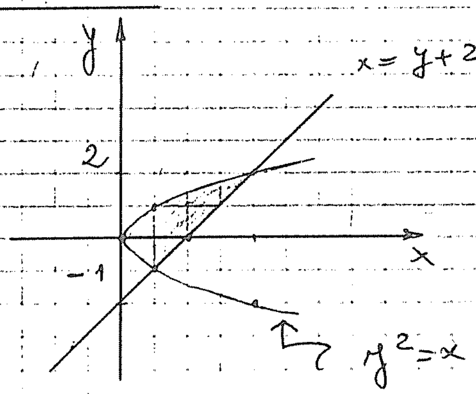
$$\text{Área} = 2\int_0^2 (4 - 2^x)\,dx = \left(4x - \frac{2^x}{\ln 2}\right)_0^2$$

$$\text{Area} = 2\left[\left(4\cdot 2 - \frac{2^2}{\ln 2}\right) - \left(4\cdot 0 - \frac{2^0}{\ln 2}\right)\right]$$

$$A = 2\left[8 - \frac{4}{\ln 2} + \frac{1}{\ln 2}\right] = 2\left[8 - \frac{3}{\ln 2}\right]$$

$$\boxed{\text{Area} = 16 - \frac{6}{\ln 2}}$$

e) $\begin{cases} y^2 = x \\ y+2 = x \end{cases}$



$$\text{Area} = \int_{-1}^{2} \left[(y+2) - y^2\right] dy$$

$$\text{Area} = \left(\frac{y^2}{2} + 2y - \frac{y^3}{3}\right)_{-1}^{2}$$

$$\text{Area} = \left(\frac{2^{\cancel{2}}}{\cancel{2}} + 2\cdot 2 - \frac{2^3}{3}\right) - \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right]$$

$$\text{Area} = \left(6 - \frac{8}{3}\right) - \left[\frac{1}{2} - 2 + \frac{1}{3}\right]$$

$$\text{Area} = \frac{10}{3} - \frac{3 - 12 + 2}{6} = \frac{10}{3} + \frac{7}{6} = \frac{20 + 7}{6}$$

$$\text{Area} = \frac{\cancel{27}^{\,9}}{\cancel{6}_{\,2}} \rightarrow \boxed{\text{Area} = \frac{9}{2}}$$

f) $\begin{cases} y = \sqrt{2(x-1)} \\ y = x-5 \\ y = 0 \end{cases}$

$(x-5)^2 = \left(\sqrt{2(x-1)}\right)^{\cancel{2}}$

$x^2 - 2.5x + 5^2 = 2x-2$

$x^2 - 10x + 25 = 2x - 2$

$x^2 - 12x + 27 = 0$

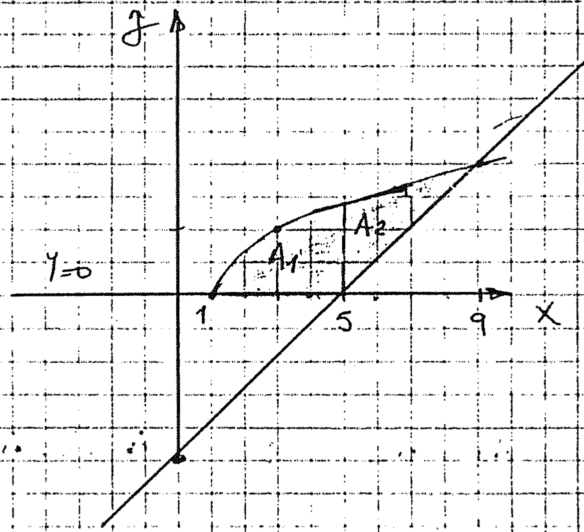
$x_{1-2} = \dfrac{12 \pm \sqrt{12^2 - 4.27}}{2}$

$x_{1-2} = \dfrac{12 \pm \sqrt{36}}{2}$

$x_{1-2} = \dfrac{12 \pm 6}{2}$

$x_1 = 9$

$x_2 = 3$



$u = 2(x-1)$

$du = 2dx$

$\int \sqrt{2(x-1)}\, dx$

$= \frac{1}{2}\int u^{1/2}\, du$

$= \frac{1}{2} \cdot \dfrac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$\dfrac{1}{\cancel{2}} \quad \dfrac{u^{3/2}}{\frac{3}{\cancel{2}}}$

$\dfrac{\sqrt{2^3(x-1)^3}}{3}$

$\text{ÁREA} = A_1 + A_2$

$\text{ÁREA} = \int_1^5 \left(\sqrt{2(x-1)} - 0\right) dx + \int_5^9 \left[\sqrt{2(x-1)} - (x-5)\right] dx$

$\text{AREA} = \int_1^5 \sqrt{2(x-1)}\, dx + \int_5^9 \left[\sqrt{2(x-1)} - x + 5\right] dx$

$\text{AREA} = \dfrac{\sqrt{8(x-1)^3}}{3}\Bigg]_1^5 + \left(\dfrac{\sqrt{8(x-1)^3}}{3} - \dfrac{x^2}{2} + 5x\right)_5^9$

$\text{AREA} = \left(\dfrac{\sqrt{8(5-1)^3}}{3} - \dfrac{\sqrt{8(1-1)}}{3}\right) +$

$+ \left(\dfrac{\sqrt{8(9-1)^3}}{3} - \dfrac{9^2}{2} + 5.9\right) - \left(\dfrac{\sqrt{8(5-1)^3}}{3} - \dfrac{5^2}{2} + 5\right.$

$\text{AREA} = \dfrac{\sqrt{8.4^3}}{3} + \dfrac{\sqrt{8^4\,{}^2}}{3} - \dfrac{81}{2} + 45$

$- \left(\dfrac{\sqrt{8.4^3}}{3} - \dfrac{25}{2} + 25\right)$

$\text{AREA} = \dfrac{64}{3} - \dfrac{81}{2} + 45 + \dfrac{25}{2} - 25$

$\boxed{\text{ÁREA} = \dfrac{40}{3}}$

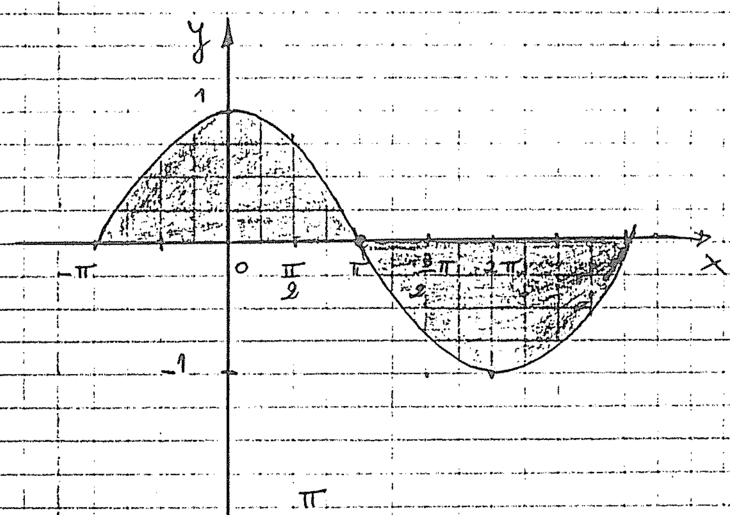
$$\text{ÁREA DE } 1/4 = \int_0^{\frac{\pi}{2}} \sqrt{r^2 - x^2}\, dx = \frac{r^2}{2}\left(t + \frac{\operatorname{sen} 2t}{2}\right)\Big]_0^{\frac{\pi}{2}}$$

$$\text{ÁREA DE } 1/4 = \frac{r^2}{2} \cdot \frac{\pi}{2} + 0$$

$$\text{ÁREA DE } 1/4 = \frac{r^2 \cdot \pi}{4}$$ esto es $\frac{1}{4}$ de círculo

$$\text{ÁREA DEL CÍRCULO} = \cancel{4}\,\frac{r^2 \cdot \pi}{\cancel{4}} = \boxed{\pi r^2}$$ área del círculo

b) Una onda de $y = \cos \frac{x}{2}$



Si $x = \pi$

$t = \frac{\pi}{2}$

$t = \frac{x}{2}$

$dt = \frac{dx}{2}$

$2\, dt = dx$

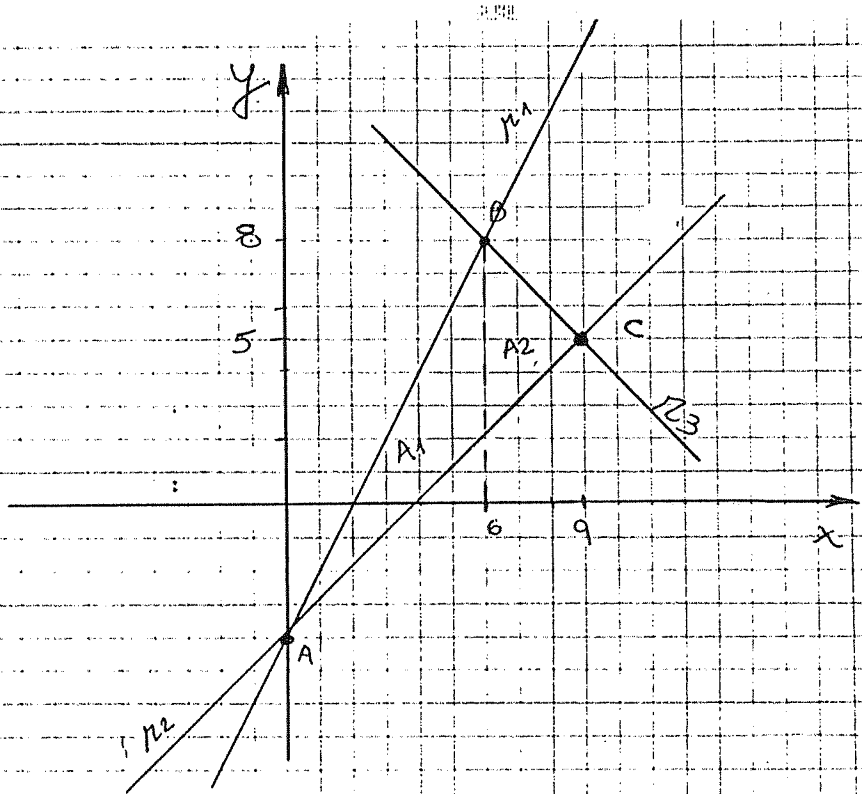
$$\text{ÁREA} = 4\int_0^{\pi} \cos \frac{x}{2}\, dx = 8 \int_0^{\frac{\pi}{2}} \cos t (2\, dt)$$

$$\text{ÁREA} = 8 \int_0^{\pi/2} \cos t\, dt$$

$$\text{ÁREA} = 8 \operatorname{sen} t \Big]_0^{\pi/2} = 8\left(\operatorname{sen} \frac{\pi}{2} - \operatorname{sen} 0\right)$$

$$\boxed{\text{ÁREA} = 8}$$

c) Un triángulo cuyos vértices son $A(0;-4)$ $B(6;8)$, $C(9;5)$



AREA $= A_1 + A_2$

$r_1 \equiv$ $A(0;-4)$, $B(6;8)$ $\Rightarrow m = \dfrac{8+4}{6-0} = \dfrac{12}{6} = \boxed{2}$

$\boxed{m_1 = 2}$

$y = m(x - x_0) + y_0$

$y = 2(x-6)+8 \Rightarrow y = 2x - 12 + 8$

$r_1 \equiv \boxed{y = 2x - 4}$

$r_2 \equiv$ $A(0;-4)$, $C(9;5)$ $\Big\}$ $m_2 = \dfrac{5+4}{9} = 1 \Rightarrow \boxed{m_2 = 1}$

$y = 1(x-0) - 4 \Rightarrow r_2 \equiv \boxed{y = x - 4}$

$r_3 \equiv$ $B(6;8)$, $C(9;5)$ $\Big\}$ $m_3 = \dfrac{5-8}{9-6} \Rightarrow m_3 = \dfrac{-3}{3} = \boxed{-1}$

$y = -1(x-6)+8 \Rightarrow y = -x + 6 + 8$

$r_3 \equiv \boxed{y = -x + 14}$

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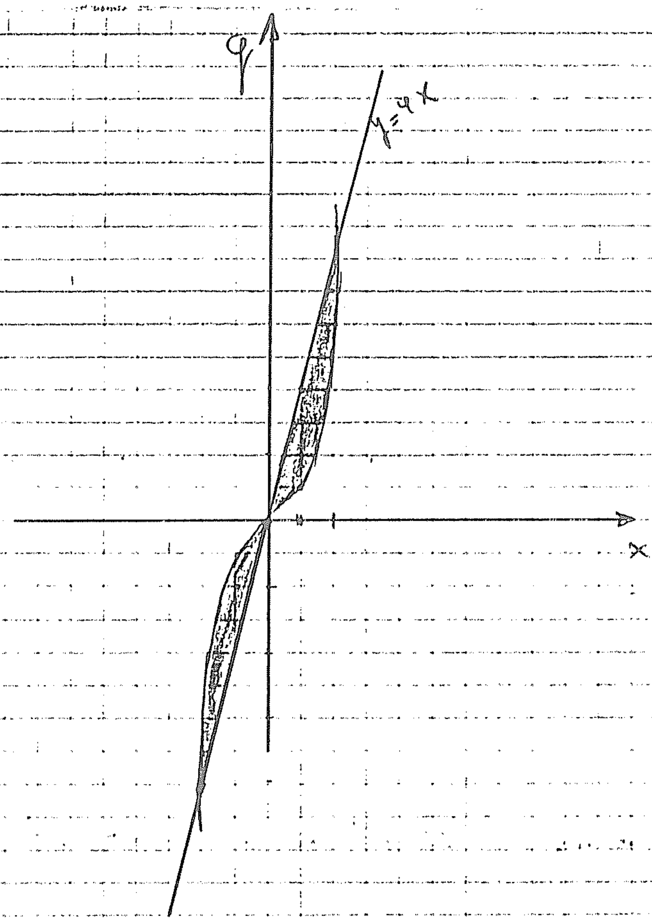
g) $\begin{cases} y = x^3 \\ y = 4x \end{cases}$



$x^3 = 4x$

$x^3 - 4x = 0$

$x \cdot (x^2 - 4) = 0$

$x = 0$

$x = 2$

$x = -2$

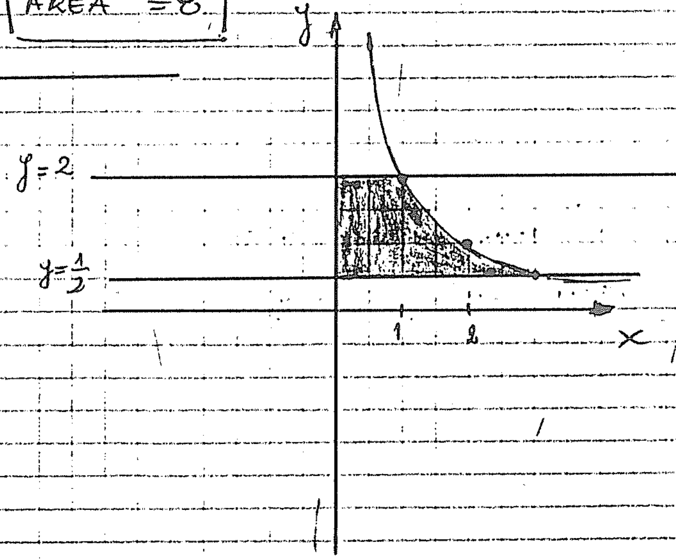
$$\text{AREA} = 2 \int_0^2 (4x - x^3)\, dx$$

$$\text{AREA} = 2 \left[\frac{\cancel{4}^{2} x^2}{\cancel{2}_{1}} - \frac{x^4}{4} \right]_0^2 = 2 \left(2 \cdot 2^2 - \frac{2^4}{4} \right)$$

$$\text{AREA} = 2\,(8 - 4) = \boxed{8}$$

$\boxed{\text{AREA} = 8}$

h) $\begin{cases} y = \dfrac{2}{x} \Rightarrow x = \dfrac{2}{y} \\ y = 2 \\ y = \dfrac{1}{2} \end{cases}$



$$\text{AREA} = \int_{\frac{1}{2}}^{2} \frac{2}{y}\, dy$$

$$\text{AREA} = 2 \int_{\frac{1}{2}}^{2} \frac{1}{y}\, dy$$

$$\text{AREA} = 2 \ln|y| \Big]_{\frac{1}{2}}^{2} = 2 \left(\ln 2 - \ln \frac{1}{2} \right) = 2 \ln 4$$

7) Hallar por integración el área a) de un círculo de radio r.

TOMO $\frac{1}{4}$ DE CÍRCULO

$$\text{ÁREA} = 4\int_0^r \sqrt{r^2-x^2}\,dx =$$

$$\text{ÁREA} = 4\left[\frac{x}{2}\sqrt{r^2-x^2} + \frac{r^2}{2}\arcsin\frac{x}{r}\right]_0^r$$

$$= 4\left[\frac{r}{2}\cdot 0 + \frac{r^2}{2}\arcsin\frac{r}{r}\right] = 4\,\frac{r^2\pi}{4} = \boxed{\pi \cdot r^2}$$

$$\boxed{\text{x TABLA: } \int \sqrt{r^2-x^2}\,dx = \frac{x}{2}\sqrt{r^2-x^2} + \frac{r^2}{2}\arcsin\frac{x}{r}}$$

SIN USAR LA TABLA

Para resolver $\int \sqrt{r^2-x^2}\,dx$ se hace una sustitución trigonométrica. $x = r \operatorname{sen} t$

$dx = r\cos t\,dt$

$$\int \sqrt{r^2-x^2}\,dx = \int \sqrt{r^2 - r^2\operatorname{sen}^2 t}\; r\cos t\,dt$$

$$= r^2 \int \underbrace{\sqrt{1-\operatorname{sen}^2 t}}_{\cos t}\,\cos t\,dt = r^2 \int \cos^2 t\,dt$$

$$= r^2 \int \frac{1}{2}(1+\cos 2t)\,dt$$

Si x varía entre 0 y r
t varía entre 0 y $\frac{\pi}{2}$

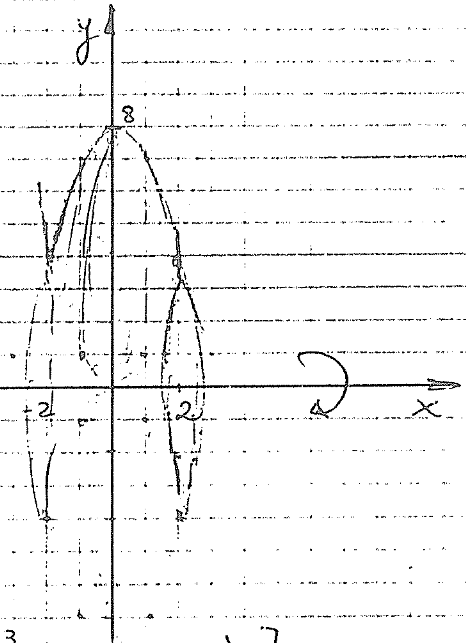
$$= \frac{r^2}{2}\left(t + \frac{\operatorname{sen} 2t}{2}\right)$$

Luego:

$$\text{ÁREA DE } \tfrac{1}{4} = \int_0^{\frac{\pi}{2}} \sqrt{r^2-x^2} = \frac{r^2}{2}\left(t + \frac{\operatorname{sen} 2t}{2}\right)\Bigg]_0 \qquad t = \arcsin\frac{x}{r}$$

$$\cos^2 t = \frac{1}{2}(1+\cos 2t)$$

b) $\begin{cases} y = x^2 \\ y = -x^2+8 \end{cases}$



$$V = \pi \int_{-2}^{2} \left[x^2 - (-x^2+8)\right] dx$$

$$V = \pi \int_{-2}^{2} (2x^2+8)\, dx$$

$$V = \pi \left[\frac{2x^3}{3} + 8x)\right]_{-2}^{2}$$

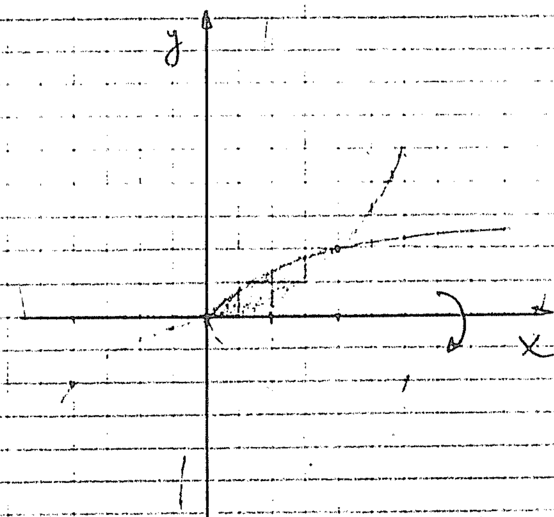
$$V = \pi \left[\left(\frac{2}{3}2^3 + 8.2\right) - \left(\frac{2}{3}(-2)^3 + 8(-2)\right)\right]$$

$$V = \pi \left[\frac{16}{3} + 16 + \frac{16}{3} + 16\right]$$

$$V = \pi \left[\frac{32}{3} + 32\right] = \frac{128}{3}\pi \text{ unidades de volumen}$$

c) $\begin{cases} y = \dfrac{x^3}{32} \\ y^2 = x \Rightarrow |y| = \sqrt{x} \end{cases}$

$$y = \frac{(y^2)^3}{32}$$



$$32y = y^6$$

$$y^6 - 32y = 0$$

$$y(y^5 - 32) = 0 \rightarrow \boxed{y=0}$$

$$y^5 - 32 = 0$$

$$y^5 = 32$$

$$y = \sqrt[5]{32}$$

$$\boxed{y = 2}$$

$$V = \pi \int_0^4 \left(\sqrt{x} - \frac{x^3}{32}\right) dx = \pi \int_0^4 \left(x^{1/2} - \frac{x^3}{32}\right) dx$$

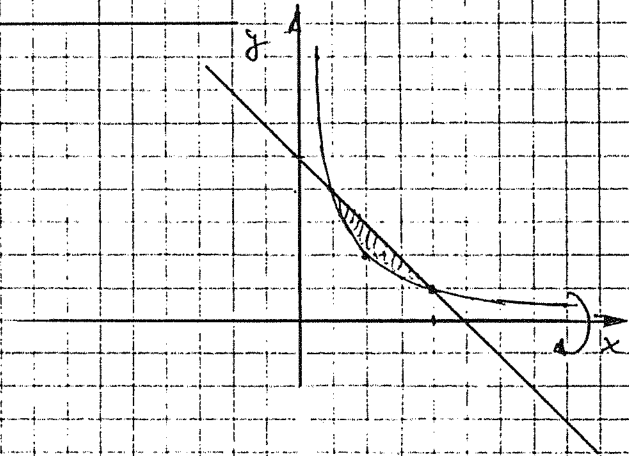
$$V = \pi \left(\frac{x^{1/2+1}}{1/2+1} - \frac{x^4}{4.32}\right)_0^4 \Rightarrow V = \pi \left(\frac{2}{3}x^{3/2} - \frac{x^4}{128}\right)_0^4$$

$$V = \pi \left(\frac{2}{3} x^{\frac{3}{2}} - \frac{x^4}{128} \right)_0^4$$

$$V = \pi \left(\frac{2}{3} \, 4^{3/2} - \frac{4^4}{128} \right) - 0$$

$$\pi \left(\frac{2}{3} \cdot 8 - 2 \right) = \boxed{\frac{10}{3} \pi \text{ u. de vol.}}$$

d) $\begin{cases} y = \frac{4}{x} \\ y = -x+5 \end{cases}$



$y = -x+5$

$\frac{x}{5} + \frac{y}{5} = 1$

$$V = \pi \int_1^4 \left[(-x+5) - 4 \frac{1}{x} \right] dx$$

$$V = \pi \left[-\frac{x^2}{2} + 5x - 4 \ln|x| \right]_1^4$$

$$V = \pi \left[\left(-\frac{4^2}{2} + 5.4 - 4 \ln 4 \right) - \left(-\frac{1^2}{2} + 5.1 - 4 \underbrace{\ln 1}_{0} \right) \right]$$

$$V = \left(-8 + 20 - 4 \ln 4 + \frac{1}{2} - 5 \right) \pi$$

$$V = \left(\frac{15}{2} - 4 \ln 4 \right) \pi \text{ unidades de volumen.}$$

---

11) Calcular la longitud del arco de curva indicada en el intervalo señalado.

Longitud de arco: $L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$

a) $y = x^{2/3}$ $[0 ; 4]$

$$y' = \frac{2}{3} x^{2/3 - 1} \Rightarrow y' = \frac{2}{3} x^{-1/3}$$

$$[f'(x)]^2 = \left(\frac{2}{3} x^{-1/3} \right)^2 = \left(\frac{2}{3} \right)^2 x^{-2/3} \Rightarrow \boxed{[f'(x)]^2 = \frac{4}{9} x^{-2/3}}$$

$$AREA = \int_0^6 \left[(2x-4) - (x-4)\right] dx + \int_6^9 \left[(-x+14) - (x-4)\right] dx$$

$$AREA = \int_0^6 (2x - x \cancel{-4} \cancel{+4})\, dx + \int_6^9 (-x + 14 - x + 4)\, dx$$

$$AREA = \int_0^6 x\, dx + \int_6^9 (-2x + 18)\, dx$$

$$A = \frac{x^2}{2}\Big]_0^6 + \left(-\frac{\cancel{2}x^2}{\cancel{2}} + 18x\right)\Big]_6^9$$

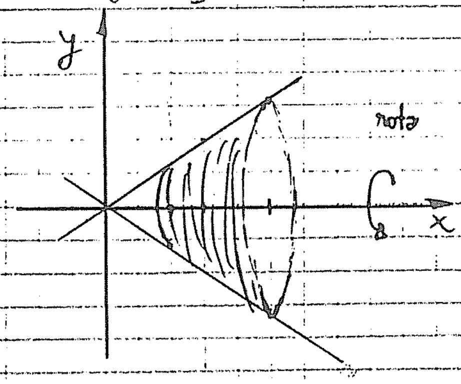
$$A = \frac{\overset{18}{\cancel{36}}}{\cancel{2}} + (-81 + 18 \cdot 9) - (-36 + 18 \cdot 6)$$

$$A = 18 - 81 + 162 + 36 - 108$$

$$\boxed{A = 27}$$

8) Hallar los volumenes de los sólidos de revolución engendrados cuando las curvas indicadas rotan alrededor del eje x.

a) $y = \frac{2}{3}x$ $[2;5]$



$$Volumen = \pi \int_a^b \left[f(x)\right]^2 dx$$

$$V = \pi \int_2^5 \left(\frac{2}{3}x\right)^2 dx$$

$$V = \frac{4}{9}\pi \int_2^5 x^2\, dx$$

$$V = \frac{4}{9}\pi \frac{x^3}{3}\Big]_2^5$$

$$V = \frac{4}{9}\pi \left[\frac{5^3}{3} - \frac{2^3}{3}\right]$$

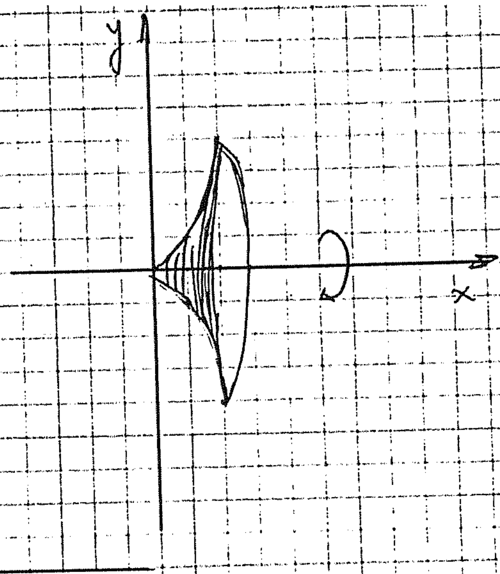
$$V = \frac{4}{9}\pi \frac{125-8}{3} \Rightarrow V = \frac{468}{27}\pi \approx \boxed{17{,}3\,\pi}$$

b) $y = x^2$ $[1;2]$

$V = \pi\int_1^2 x^4\,dx$

$V = \pi \left.\frac{x^5}{5}\right]_1^2 = \pi\left(\frac{32}{5}-\frac{1}{5}\right)$

$\boxed{V = \frac{31}{5}\pi}$ u. de v.



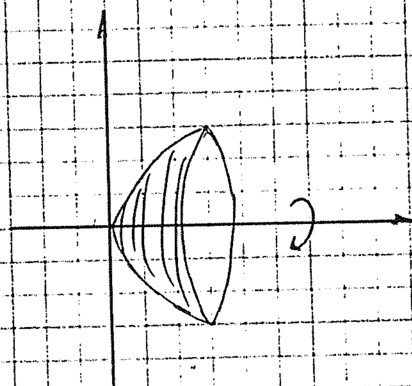
c) $y = \operatorname{sen} x$ $\left[0, \frac{\pi}{2}\right]$

$V = \pi\int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x\,dx$

$V = \pi\int_0^{\frac{\pi}{2}} \frac{1}{2}\,(1-\cos 2x)\,dx$

$V = \pi\,\frac{1}{2}\left.\left(x - \frac{\operatorname{sen} 2x}{2}\right)\right]_0^{\frac{\pi}{2}}$

$V = \frac{\pi}{2}\left[\frac{\pi}{2} - \frac{\overbrace{\operatorname{sen}\cancel{2}\pi/\cancel{2}}^{0}}{2}\right] \Rightarrow \boxed{V = \frac{\pi^2}{4}}$ unidades de volumen



d) $y = x^2+3$ $[-2;2]$

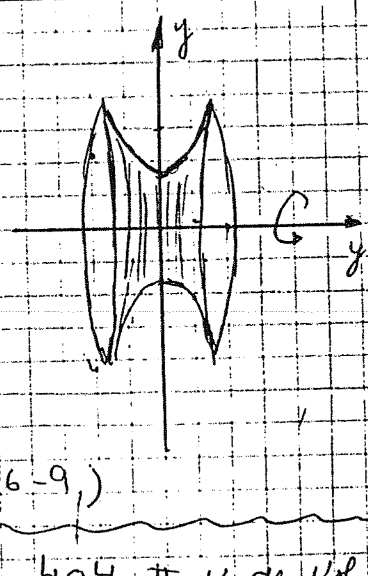
$V = \pi\int (x^2+3)^2\,dx$

$V = \pi\int_{-2}^{2} (x^4+6x^2+9)\,dx$

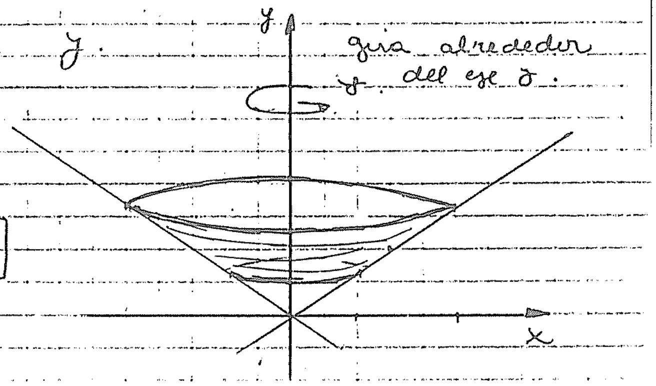
$V = \pi\left.\left(\frac{x^5}{5} + \frac{\overset{2}{\cancel{6}}x^3}{\underset{1}{\cancel{3}}} + 9x\right)\right]_{-2}^{2}$

$V = \pi\left[\left(\frac{32}{5} + 16 + 18\right) - \left(-\frac{32}{5} - 16 - 9\right)\right]$

$V = \pi\,\frac{202}{5}\cdot 2 \Rightarrow \boxed{V = \frac{404}{5}\pi \text{ u. de vol.}}$



9) Resolver el ejercicio anterior cuando la curva rota alrededor del eje y.



a) $y = \frac{2}{3}x \quad [2;5]$

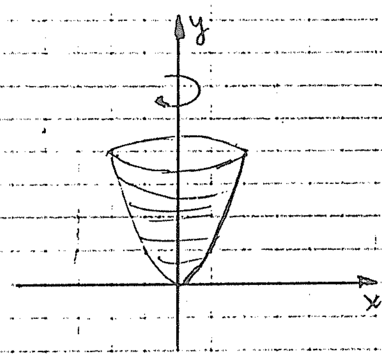
$x = \frac{3}{2}y \quad \left[\frac{4}{3};\frac{10}{3}\right]$

$V = \pi \int \left(\frac{3}{2}y\right)^2 dy$

$V = \pi \frac{9}{4} \int_{4/3}^{10/3} y^2\, dy = \frac{9}{4}\pi \left.\frac{y^3}{3}\right]_{\frac{4}{3}}^{\frac{10}{3}}$

$V = \frac{9}{4}\pi \frac{1}{3}\left(\frac{10^3}{3^3} - \frac{4^3}{3^3}\right) \Rightarrow V = \frac{9}{4}\pi \cdot \frac{1}{81}\left(10^3 - 4^3\right)$

$V = \frac{2}{9}\pi \cdot 117$



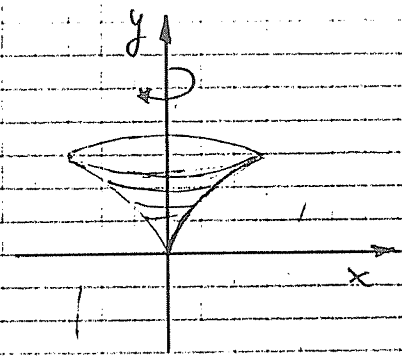
b) $y = x^2 \quad [1;2]$

$x = \sqrt{y} \quad [1;4]$

$V = \pi \int_1^4 (\cancel{\sqrt{y}})^{\cancel{2}}\, dy$

$V = \pi \int_1^4 y\, dy \Rightarrow V = \pi \left.\frac{y^2}{2}\right]_1^4$

$V = \pi\left(\frac{16}{2} - \frac{1}{2}\right) \Rightarrow \boxed{V = \frac{15}{2}\pi}$



c) $y = \operatorname{sen} x \quad \left[0;\frac{\pi}{2}\right]$

$x = \operatorname{arcsen} y$

$y = \operatorname{sen} 0 = 0$

$y = \operatorname{sen} \frac{\pi}{2} = 1$

$V = \pi \int_0^1 (\operatorname{arcsen} y)^2\, dy$

x TABLA

$$\int \left(\arcsin \frac{x}{a}\right)^2 dx = x\left(\arcsin\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\ \text{arc}$$

$$V=\pi\int_0^1 \arcsin^2 y\, dy = \pi\left(y \arcsin^2 y - 2y - 2\sqrt{\ ^2-y^2}\ \arcsin y\right)\Big]_0^1$$

$$V=\pi\left\{\left[1\arcsin^2 1 - 2\cdot 1 - 2\sqrt{1-1^2}\ \arcsin 1\right] - \left[0-0-2\cdot 1\arcsin 0\right]\right.$$

$$V=\pi\left\{\left[\frac{\pi}{2} - 2 - \overbrace{\frac{2\,0\,\pi}{2}}^{0}\right] - 0\right\}$$

$$V = \pi\left(\frac{\pi}{2}-2\right) = \left(\frac{\pi^2}{2} - 2\pi\right)$$ unidades de vol.

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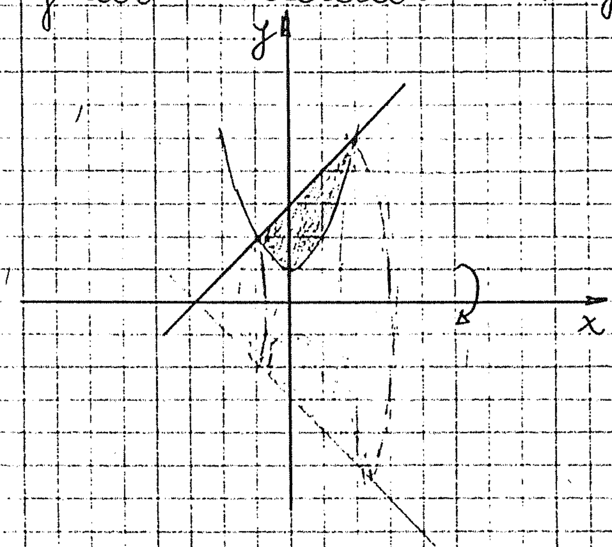
10) Calcular el volumen del cuerpo engendrado por la figura limitada por las curvas que se indican al girar alrededor del eje x.

a) $\begin{cases} y = x^2+1 \\ y = x+3 \end{cases}$



$$x^2+1 = x+3$$

$$x^2-x-2=0$$

$$x_{1-2} = \frac{-(-1)\pm\sqrt{(-1)^2-4\cdot(-2)}}{2}$$

$$x_{1-2} = \frac{1\pm\sqrt{1+8}}{2}$$

$$x_{1-2} = \frac{1\pm 3}{2} \nearrow x_1 = \frac{1+3}{2} = \boxed{2} \qquad \searrow x_2 = \frac{1-3}{2} = \boxed{-1}$$

$$V = \pi\int_{-1}^{2}\left[(x+3)-(x^2+1)\right] dx$$

$$V = \pi\int_{-1}^{2}(-x^2+x+2)\, dx = \pi\left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)\Big]_{-1}^{2}$$

$$V = \pi\left[\left(-\frac{8}{3}+2+4\right) - \left(-\frac{1}{3}+\frac{1}{2}-2\right)\right] = \boxed{\frac{3}{2}\pi}$$ unidades de vol.