

$$I_{cm} = \frac{2}{5} m R^2$$

HOJA Nº

FECHA

41) PÁG. 37

$$m = 1000 \text{ kg}$$

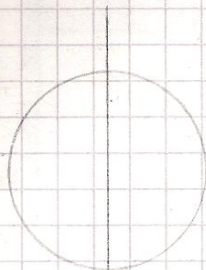
$$L = I \cdot \omega$$

$$r = 0,5 \text{ m}$$

$$\omega_0 = 36 \text{ rpm} (3,76 \text{ rad/s})$$

$$E_c = ?$$

$$L = ?$$



$$E_{ch} = \frac{1}{2} I \omega_0^2 \Rightarrow E_{ch} = \frac{1}{2} \left(\frac{2}{5} m R^2 \omega_0^2 \right) \Rightarrow E_{ch} = 706,5 \text{ J}$$

$$\frac{1}{2} 400 \cdot 0,25 \cdot 14,13$$

$$L = I \omega \Rightarrow \frac{2}{5} m R^2 \omega_0 = L \Rightarrow L = 376 \text{ kg m}^2/\text{s}$$

42) PÁG. 37

$$r = 1 \text{ m}$$

$$\Delta \theta = 62,83 \text{ rad}$$

$$P = 981 \text{ N} (100 \text{ kg})$$

$$M = 10 \text{ kg f} \cdot \text{m}$$

$$N = 10 \text{ VUELTAS}$$



$$10 \cdot \frac{10}{2} = \frac{1}{2} m R^2 \cdot \epsilon$$

$$\frac{10 \text{ kg f} \cdot \text{m}}{\frac{1}{2} 100} = \epsilon \Rightarrow \epsilon = 2 \frac{1}{5} \text{ rad/s}^2$$

$$\omega^2 = \omega_0^2 + 2 \epsilon \Delta \theta$$

$$\omega_f = \sqrt{2 \epsilon \Delta \theta} \Rightarrow \omega_f = 15,85 \text{ rad/s}$$

$$E_{ch} = \frac{1}{2} I \cdot \omega^2 \Rightarrow \frac{1}{2} \left(\frac{1}{2} m R^2 \right) \omega^2 \Rightarrow E_{ch} = 25 \text{ kg} \cdot 1 \text{ m}^2 \cdot 251,22 \frac{1}{\text{s}^2}$$

$$E_{ch} =$$

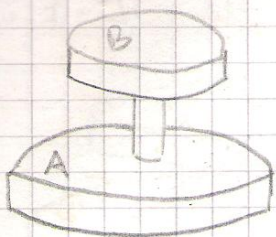
$$W = M \cdot \Delta \theta \Rightarrow W = 10 \text{ kg f} \cdot \text{m} \cdot 62,83 \text{ rad} \Rightarrow W = 628,3 \text{ kg f} \cdot \text{m}$$

~~ESTA BIEN LA FORMULA?~~

SI.

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PÁG. 37.



$$\omega_A = 400 \text{ rpm} \left(41,88 \frac{1}{s} \right) \quad R_A = 0,15 \text{ m}$$

$$m_A = 4 \text{ kg} \quad / m_B = 1,77 \text{ kg} \quad R_B = 0,10 \text{ m}$$

$$\omega_f = ?$$

$$\Delta E_c = ?$$

$$p_A = \frac{m_A}{V_A}, \quad p_B = \frac{m_B}{V_B}$$

$$\Rightarrow p_A = p_B$$

$$\Rightarrow \frac{m_A}{V_A} = \frac{m_B}{V_B}$$

$$\Rightarrow m_B = \frac{m_A \cdot V_B}{V_A}$$

$$m_B = \frac{m_A \cdot V_B}{V_A} \Rightarrow m_B = \frac{4 \text{ kg} \cdot \pi \cdot R_B^2}{\pi \cdot R_A^2} \Rightarrow m_B = 1,77 \text{ kg}$$

$$L_0 = \frac{1}{2} m_A R_A^2 \cdot \omega_0$$

$$L_f = \frac{1}{2} m_A R_A^2 \cdot \omega_f + \frac{1}{2} m_B R_B^2 \cdot \omega_f$$

$$L_0 = L_f \Rightarrow \frac{1}{2} m_A R_A^2 \omega_0 = \omega_f \left(\frac{1}{2} m_A R_A^2 + \frac{1}{2} m_B R_B^2 \right)$$

$$\omega_f = \frac{\frac{1}{2} m_A R_A^2}{\frac{1}{2} m_A R_A^2 + \frac{1}{2} m_B R_B^2} \cdot \omega_0 \Rightarrow \omega_f = \frac{0,045 \text{ kg/m}^2 \cdot 41,88 \frac{1}{s}}{0,045 + 0,00885 \text{ kg/m}^2}$$

$$\omega_f = 34,99 \frac{1}{s}$$

$$I_T = \frac{1}{2} m_A R_A^2 + \frac{1}{2} m_B R_B^2$$

$$I_T = 0,05385$$

$$\Delta E_c = E_{cf} - E_{ci} \Rightarrow E_{ci} = \frac{1}{2} \left(\frac{1}{2} m_A R_A^2 \right) (\omega_0)^2 \Rightarrow E_{ci} = 39,46 \text{ J}$$

$$E_{cf} = \frac{1}{2} \left[\frac{1}{2} (m_A R_A^2 + m_B R_B^2) \cdot \omega_f^2 \right] \Rightarrow E_{cf} = 32,96 \text{ J}$$

$$\Delta E_c = E_{cf} - E_{ci} \Rightarrow \Delta E_c = 32,96 \text{ J} - 39,46 \text{ J} \Rightarrow \Delta E_c = -6,5 \text{ J}$$

$$E_{CT} = \frac{1}{2} m_1 v^2$$

$$E_{CF} = \frac{1}{2} I \omega^2$$

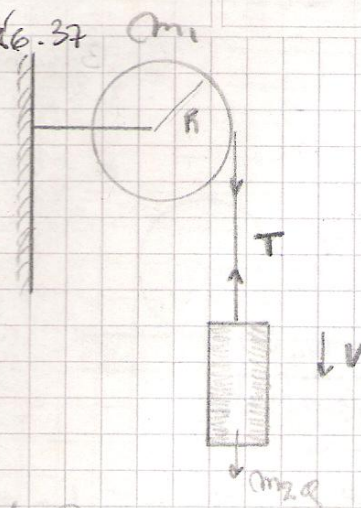
$$\frac{a}{R} = \epsilon$$

HOJA N°

FECHA

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$$m_1 = m_2 = 20 \text{ kg} \quad (196 \text{ N})$$



$$m_2 g - T = m_2 a$$

$$T = m_2 (g - a) \quad (2)$$

$$T \cdot R = \frac{1}{2} m_1 R^2 \cdot \frac{a}{R}$$

$$T = \frac{1}{2} m_1 \cdot a \quad (1)$$

Ecuación (1)...

$$T = \frac{1}{2} m_1 a$$

$$m_2 (g - a) = \frac{1}{2} m_1 a \Rightarrow g = \frac{3}{2} a \Rightarrow a = \frac{2}{3} g \rightarrow 6,53 \text{ m/s}^2$$

$$v = a t \Rightarrow v = 6,53 \text{ m/s}^2 \cdot 10 \text{ s} \Rightarrow v = 65,3 \text{ m/s}$$

$$v = v_0 + at$$

$$\omega = \frac{v}{r}$$

$$E_{CF} = \frac{1}{2} m_2 v^2 + \frac{1}{2} I \cdot \omega^2$$

$$E_{CF} = 42640 \text{ J} + \frac{1}{4} m_1 R^2 \frac{v^2}{R^2} \Rightarrow E_{CF} = 42640 \text{ J} + 21320 \text{ J}$$

$$E_{CF} = 63960 \text{ J} \quad \text{ó} \quad 64 \text{ kJ} \quad (\text{como dice la Guía})$$

45 PAG. 37

$$r = 0,04 \text{ m (4 cm)} \quad \Delta y = 3 \text{ m}$$

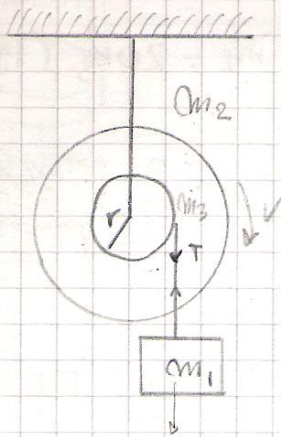
$$\Delta t = 8 \text{ s}$$

$$m_1 = 2 \text{ kg}$$

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$3 = \frac{1}{2} t^2 a$$

$$a = 0,093 \text{ m/s}^2$$



m_1 / TRASLACIÓN

$$m_1 g - T = m_1 a$$

$$T = m_1 (g - a) \quad (1)$$

m_2 / ROTACIÓN

$$\Sigma \tau = I \cdot \epsilon$$

$$r \cdot T = I \cdot \frac{a}{r} \quad (2)$$

$$\frac{r^2 \cdot T}{a} = I \Rightarrow \frac{(0,04 \text{ m})^2 \cdot m_1 (g - a)}{a} = I$$

$$\frac{1,6 \times 10^{-3} \text{ m}^2 \cdot 2 \text{ kg} (9,707 \text{ m/s}^2)}{0,093 \text{ m/s}^2} = I \Rightarrow \boxed{I = 0,334 \text{ kg m}^2}$$

b) $E_{ch} = ? \Rightarrow E_{ch} = \frac{1}{2} I \omega_F^2$

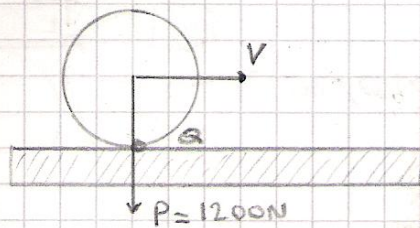
$$V_f = V_0 + a \cdot t \Rightarrow V_f = a \cdot t \Rightarrow V_f = 0,093 \text{ m/s}^2 \cdot 5 \text{ s} \Rightarrow V = 0,465 \text{ m/s}$$

$$E_{ch} = \frac{1}{2} I \cdot \left(\frac{V_f}{r} \right)^2 \Rightarrow E_{ch} = 0,167 \text{ kg m}^2 \cdot \frac{0,216 \text{ m}^2/\text{s}^2}{1,6 \times 10^{-3} \text{ m}^2} \Rightarrow \boxed{E_{ch} = 22,54 \text{ J}}$$

46) PÁG. 38

$$P = 1200 \text{ N} (120 \text{ kg})$$

$$V = 10 \text{ m/s}$$



$$E_{ch} = \frac{1}{2} I \omega^2 \Rightarrow E_{ch} = \frac{1}{2} \left(\frac{3}{2} m R^2 \right) \frac{V^2}{R^2} \Rightarrow E_{ch} = \frac{3}{4} \cdot 120 \text{ kg} \cdot 100 \frac{\text{m}^2}{\text{s}^2}$$

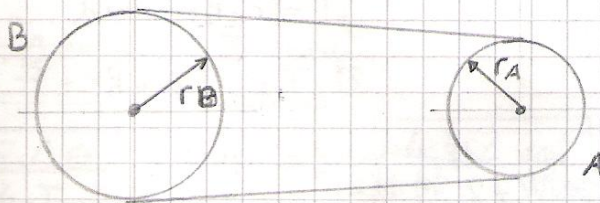
$$E_{ch} = 9000 \text{ J} \quad \text{con } g = 10 \text{ m/s}^2$$

$$E_{ch} = 9183 \text{ J} \quad \text{con } g = 9,8 \text{ m/s}^2 \text{ (cambia la masa)}$$

48) PÁG. 38

$$r_A$$

$$r_B = 3r_A$$



$$L = I \omega$$

$$L_A = L_B$$

$$L_A = I_A \cdot \frac{V}{r_A} ; L_B = I_B \cdot \frac{V}{3r_A}$$

$$I_A \cdot \frac{V}{r_A} = I_B \cdot \frac{V}{3r_A}$$

$$\frac{I_A}{I_B} = \frac{r_A}{3r_A} \Rightarrow \frac{I_A}{I_B} = \frac{1}{3}$$

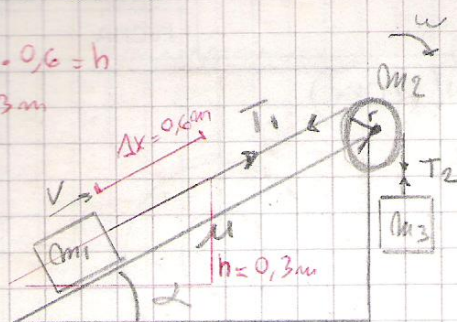
$$E_{ch} = \frac{1}{2} I \cdot \omega$$

$$\frac{1}{2} I_A \frac{V^2}{r_A^2} = \frac{1}{2} I_B \frac{V^2}{9r_A^2}$$

$$\frac{I_A}{I_B} = \frac{r_A^2}{9r_A^2} \Rightarrow \frac{I_A}{I_B} = \frac{1}{9} \quad \checkmark$$

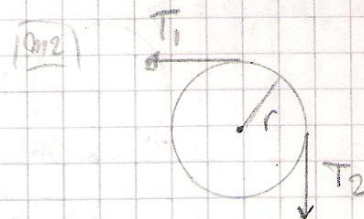
50 PÁG. 38

$\sin 30^\circ \cdot 0,6 = h$
 $h = 0,3 \text{ m}$



$m_1 = 50 \text{ g} (0,05 \text{ kg})$
 $m_2 = 400 \text{ g} (0,4 \text{ kg})$
 $m_3 = 600 \text{ g} (0,6 \text{ kg})$
 $\Delta y = 0,6 \text{ m}$
 $\mu = 0,2$
 $\alpha = 30^\circ$

ROTACION



a) ECUACION 1

$$T_2 \cdot R - T_1 \cdot R = \frac{1}{2} m_2 R^2 \frac{a}{R}$$

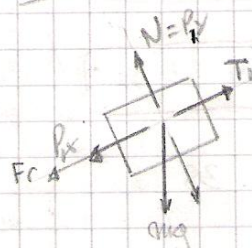
1) $T_2 \cdot R - T_1 \cdot R = I \cdot \epsilon$

$$m_3 g - m_3 a - [m_1 a + m_1 g \sin \alpha + \mu \cdot m_1 g \cos \alpha] = \frac{1}{2} m_2 a$$

$$\frac{m_3 g - m_1 g \sin \alpha - \mu m_1 g \cos \alpha}{\frac{1}{2} m_2 + m_3 + m_1} = a$$

$$\frac{5880 \text{ N} - 245 \text{ N} - 84,87 \text{ N}}{850 \text{ kg}} = a \Rightarrow a = 6,52 \text{ m/s}^2$$

m1 TRANSLACION



$P_x = m_1 g \sin \alpha$

$P_y = m_1 g \cos \alpha$

$Fr = P_y \cdot \mu$

$T - Fr - P_x = m_1 a$

2) $T_1 = m_1 a + P_x + Fr$

$P_x = 245 \text{ N}$

$P_y = 424,3 \text{ N}$

$Fr = 84,87$

$P_x = 0,245 \text{ N}$

$P_y = 0,42$

$Fr = 0,084$

b) $T_1 = m_1 a + P_x + Fr \Rightarrow T_1 = 0,655 \text{ N}$

$T_2 = m_3 (g - a) \Rightarrow T_2 = 1,96 \text{ N}$

c) $V_f^2 = V_0^2 + 2 a \Delta y \Rightarrow V_f = \sqrt{2 a \Delta y} \Rightarrow V_f = 2,79 \text{ m/s}$

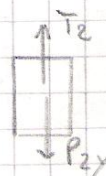
$W_{FNC} = \Delta E_m$

$-Fr \cdot \Delta x = \Delta E_m$

$-(0,084 \text{ N}) \cdot 0,6 \text{ m} = \Delta E_m$

$\Delta E_m = -0,051 \text{ J}$

m3



$P_{2y} - T_2 = m_3 a$

3) $T_2 = m_3 (g - a)$

$$\frac{1}{2} I \omega^2$$

HOJA N° 24/22

FECHA

SI PAG. 38

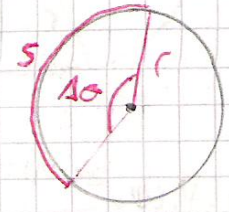
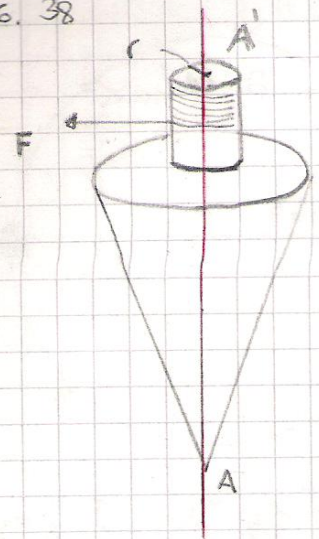
$$I_T = 4 \cdot 10^{-4} \text{ kg m}^2$$

$$F = 5,57 \text{ N}$$

$$\Delta \theta = 8 \text{ rad} (0,8 \text{ m})$$

$$S = \Delta \theta \cdot r$$

$$\frac{S}{r} = \Delta \theta$$



$$M = I \cdot E$$

$$F \cdot \Delta \theta = I \cdot$$

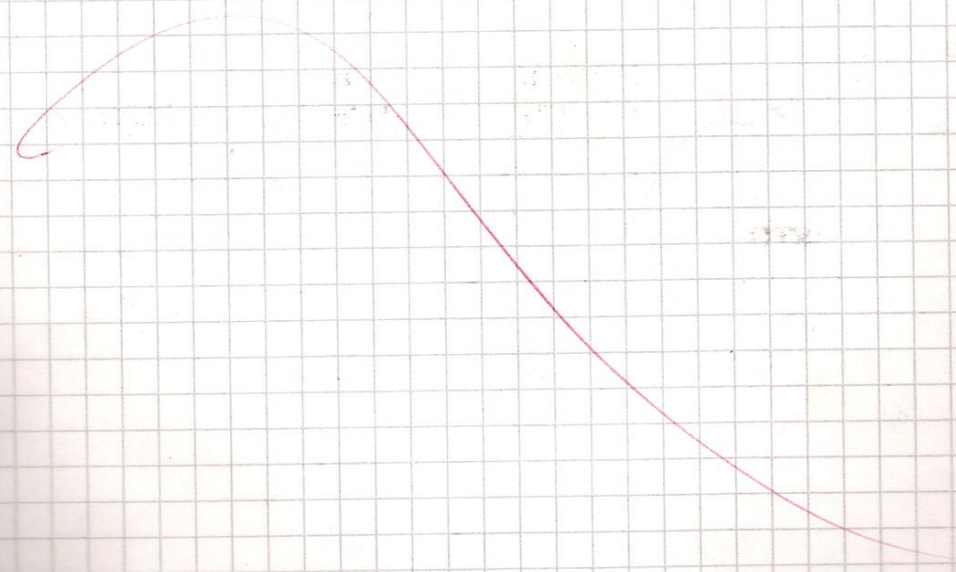
$$W_{FNC} = \Delta E_m$$

$$E_{mo} = mgh + \frac{1}{2} m v_o^2 + \frac{1}{2} I \omega_o^2 \Rightarrow E_{mo} = 0$$

$$E_{mf} = mgh + \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 \Rightarrow E_{mf} = \frac{1}{2} I \omega_f^2$$

$$F \cdot \Delta \theta = \frac{1}{2} I \cdot \omega_f^2$$

$$\sqrt{\frac{F \cdot \Delta \theta}{\frac{1}{2} I}} = \omega_f \Rightarrow \sqrt{\frac{4,456 \text{ Nm}}{2 \cdot 10^{-4} \text{ kg m}^2}} = \omega \Rightarrow 149,26 \frac{\text{kg m/s}^2}{\text{kg m}} \Rightarrow \omega_f = 149,26 \frac{1}{s}$$



$$M_P = 100 \text{ kg}$$

$$R_P = 2 \text{ m}$$

$$\omega_0 = 2 \text{ rad/s}$$

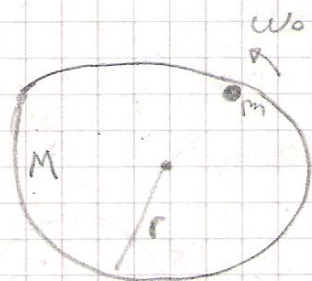
$$r = 0,5 \text{ m}$$

$$m_J = 60 \text{ kg}$$

a) $\omega_{\text{scm}} =$

$$a = \epsilon \cdot R$$

$$a = \omega^2 \cdot r$$



a)

$$L = I \cdot \omega$$

$$L_0 = (I_P + I_J) \cdot \omega_0 ; L_F = (I_P + I_J) \cdot \omega_F$$

$$L_0 = L_F$$

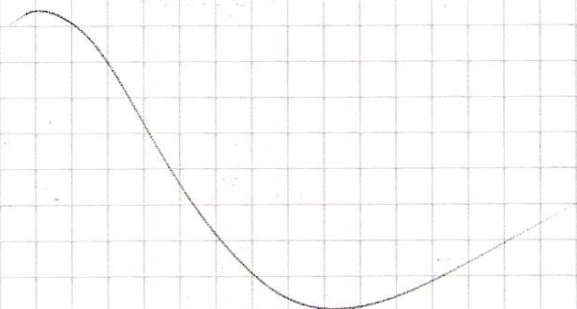
$$(I_P + I_J) \omega_0 = (I_P + I_J) \cdot \omega_F \Rightarrow \frac{\frac{1}{2} M_P R_P^2 + m_J R^2}{\frac{1}{2} M_P R_P^2 + m_J r^2} \cdot \omega_0 = \omega_F$$

$$\Rightarrow \omega_F = \frac{(200 + 240) \cdot 2}{200 + 15} = \boxed{\omega_F = 4,09} / \frac{\frac{1}{2} M_P R_P^2 + m_J r^2}$$

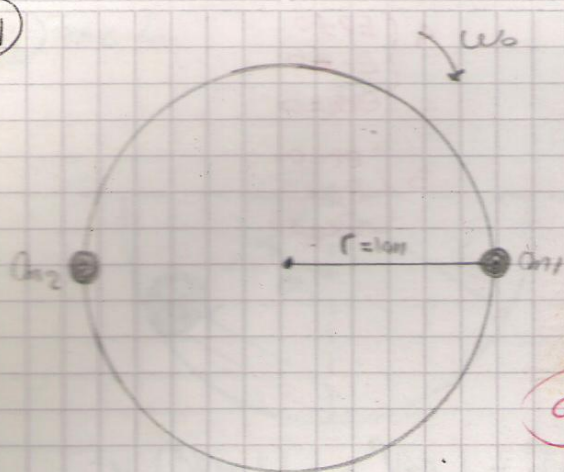
b) $E_{Ch} = \frac{1}{2} (I_P + I_J) \cdot \omega_0^2 \Rightarrow E_{Ch} = \frac{1}{2} \left(\frac{1}{2} M_P R_P^2 + m_J R^2 \right) \omega_0^2 \Rightarrow \boxed{E_{Ch} = 880 \text{ J}}$

$$E_{ChF} = \frac{1}{2} (I_P + I_J) \omega_F^2 \Rightarrow E_{Ch} = \frac{1}{2} \left(\frac{1}{2} M_P R_P^2 + m_J r^2 \right) \omega_F^2 \Rightarrow \boxed{E_{Ch} = 1798 \text{ J}}$$

c) EL INCREMENTO DE ENERGÍA ESTÁ DADO POR LA DISMINUCIÓN DEL RADIO, LA CUAL HACE QUE AUMENTE LA VELOCIDAD ANGULAR.



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$$m_1 = m_2 = 30 \text{ kg}$$

$$R = 1 \text{ m}$$

$$\mu = 0$$

$$\omega_0 = 0,5 \text{ rpm} \left(0,052 \frac{1}{s} \right)$$

$$r_1 = 0,5 \text{ m}$$

$$L = I \cdot \omega$$

$$L = (20 \text{ m} \cdot \text{h}^2) \cdot \omega_0$$

$$a) \quad L = 3,12 \text{ kg m}^2/\text{s}$$

$$b) \quad E_{ch_i} = \frac{1}{2} I \cdot \omega_0^2 \Rightarrow E_{ch_i} = \frac{1}{2} (20 \text{ m} \cdot \text{h}^2) \omega_0^2 \Rightarrow E_{ch_i} = 0,081 \text{ J}$$

$$81,2 \cdot 10^{-3} \text{ J}$$

$$c) \quad L_0 = I_0 \omega_0 \Rightarrow L_0 = (20 \text{ m} \cdot \text{h}^2) \cdot \omega_0$$

$$L_f = I_f \omega_f \Rightarrow L_f = (20 \text{ m} \cdot r_1^2) \omega_f$$

$$L_0 = L_f \Rightarrow \frac{(20 \text{ m} \cdot \text{h}^2)}{(20 \text{ m} \cdot r_1^2)} \cdot \omega_0 = \omega_f \Rightarrow \omega_f = 0,208 \frac{1}{s}$$

$$E_{ch_f} = \frac{1}{2} I_f \cdot \omega_f^2 \Rightarrow E_{ch_f} = \frac{1}{2} (20 \text{ m} \cdot r_1^2) \omega_f^2 \Rightarrow E_{ch_f} = 0,324 \text{ J}$$

d) LA VARIACIÓN DE LA ECH ESTÁ DADA POR LA DISMINUCIÓN DEL RADIO, LA CUAL PRODUCE UN AUMENTO EN LA VELOCIDAD ANGULAR